

Student Workbook

Rational Functions

Name: _____

Date: _____

Period: _____

Learning Target: I can find the domain, asymptotes, and holes of a rational function.

Vocabulary

Term	Meaning
Rational Function	$f(x) = p(x)/q(x)$, a ratio of two polynomials.
Vertical Asymptote	A vertical line the graph approaches but never touches, where the (simplified) denominator = 0.
Hole	A single missing point on the graph, caused by a common factor that CANCELS from numerator and denominator.
Horizontal Asymptote	A horizontal line the graph approaches as $x \rightarrow \pm\infty$.
Slant (Oblique) Asymptote	A slanted line the graph approaches, occurring when the numerator's degree is exactly one more than the denominator's.

Key Concept: Holes vs. Vertical Asymptotes

FACTOR FIRST! Always factor the numerator and denominator before deciding. If a factor CANCELS, that x-value is a HOLE. If a factor remains only in the denominator, that x-value is a VERTICAL ASYMPTOTE.

EXAMPLE 1
 $f(x) = (x^2-4)/(x-2) = [(x-2)(x+2)]/(x-2)$
The (x-2) factor cancels → **hole** at x = _____. Simplified form: $f(x) =$ _____

Key Concept: Horizontal Asymptotes

COMPARE THE DEGREES

Comparison	Horizontal Asymptote
$\text{degree}(\text{num}) < \text{degree}(\text{denom})$	$y = 0$
$\text{degree}(\text{num}) = \text{degree}(\text{denom})$	$y = \text{ratio of leading coefficients}$
$\text{degree}(\text{num}) > \text{degree}(\text{denom})$	none (may have a slant asymptote instead)

EXAMPLE 2

$f(x) = (2x^2+3)/(x^2-1)$ — degrees are equal (2 and 2).

Horizontal asymptote: $y = \underline{\hspace{2cm}}$

Key Concept: Slant Asymptotes

COMMON MISTAKE

Don't forget to factor and check for holes FIRST. A vertical asymptote only exists where the factor does NOT cancel.

EXAMPLE 3

$f(x) = (x^2-1)/(x+2)$. Since $\text{degree}(\text{num})$ is exactly 1 more than $\text{degree}(\text{denom})$, divide to find the slant asymptote.

$x^2-1 \div (x+2)$: quotient = $x-2$, remainder = 3

Slant asymptote: $y = \underline{\hspace{2cm}}$

Guided Practice — Part A: Domain, Vertical Asymptotes & Holes

A1 $f(x) = (x+5)/(x^2-25)$

A2 $f(x) = (x^2-9)/(x-3)$

Guided Practice — Part B: Horizontal Asymptote

B1 $f(x) = (3x^2+2)/(x^2-4)$

B2 $f(x) = (5x+1)/(x^2+3)$

Guided Practice — Part C: Slant Asymptote

C1 $f(x) = (x^2+3x+1)/(x+1)$

C2 $f(x) = (2x^2-5)/(x+3)$

Guided Practice — Part D: Full Analysis

D1 $f(x) = (x^2-1)/(x^2-4)$: find domain, VA, holes, HA.

D2 $f(x) = (x-3)/(x^2-9)$: find domain, VA, holes, HA.

BEFORE YOU MOVE ON

Check with a partner: did you factor BOTH the numerator and denominator before identifying holes and vertical asymptotes?